Spontaneous symmetry breaking in a system of strongly interacting multicomponent fermions (electrons with spin and conducting nanotubes)

V. V. Afonin, V. L. Gurevich
A. F. Ioffe Institute of the Russian Academy of Sciences, 194021 Saint Petersburg, Russia

V. Yu. Petrov
Theory Division, Saint Petersburg, Nuclear Physics Institute of the Russian Academy of Sciences

We calculate the ground state wave functions for a systems of multicomponent interacting fermions. We show that it describes the state with spontaneously broken chiral symmetry. In the limit of an infinitely strong interaction it turns into a phase with a finite density of chiral complexes. The number of particles constituting a complex depends on the number of fermion components. For example, in the case of two component electrons (spin) the condensate is built of four-particle complexes consisting of two ,,right" electrons and two ,,left" holes with the opposite spins.

PACS numbers: 71.10.Hf, 71.10.Pm., 73.63.Fg

I. INTRODUCTION AND DISCUSSION OF RESULTS

Advances in semiconductor technology have renewed interest in the properties of one-dimensional (1D) electron systems. It is well known that the electron-electron (e-e) interaction alters properties of 1D system qualitatively. To gain a better understanding of the problem one should try to clarify the physical question of a primary importance: what is the nature of the ground state of this system? In order to get the answer one usually studies the ground state for 1D interacting fermions using the ,,density-density" correlation functions. However this information is not direct. As it has been shown the correlation functions of the problem contain the terms that decay very slowly. Usually they are interpreted in the following way: the oscillations with the Fermi momentum (p_F) doubled are related to the Peierls instability (Refs.[1],[2]) while the oscillations behaving as $4p_F$ are interpreted as a marginal Wigner crystal[3].

In Ref.[4] the wave function of the ground state of spinless fermions was constructed for the exactly soluble Tomonaga – Luttinger model. It has been shown that at sufficiently low temperatures the system should be in the state that has nothing in common with a system undergoing the Peierls transition. It is a state with a spontaneously broken chiral symmetry. There is a long-range order in the electron system. In the limit of infinitely strong interaction, at low temperatures a condensate of finite density is formed. It consists of neutral (exciton-like) pairs of a right electron and a left hole or *vice versa*. The uniqueness of a one dimensional (1D) system impels one to consider a phase transition of the 2nd kind in a channel of a finite length $L_\parallel$. The point is that the temperature of the phase transition vanishes as $1/L_\parallel$ as it should be. Note that actually the phase transition temperature need not be too small. For the length $L_\parallel \sim 10^{-4}$ cm it should be about 1 K. In other words, consideration of the limit $L_\parallel \to \infty$ cannot be sufficient for description of the modern experiment.

Traditionally many-component fermions in the 1D systems have been extensively discussed in the literature. The interest was going to rouse by separation the spatial and spin degrees of freedom[1],[2]. In the present paper we are going to discuss the form of the ground state wave function for this case. As the variables are separable one can expect that the ground state wave function is a direct product of two factors where one of the factors, describing the spinless component, coincides with the ground state wave function of spinless fermions. In the present paper we show that in fact one has an entirely different situation. Namely, the most correlated state is the state where the total spin vanishes. However, for n-component electrons this state consists not of pairs (as in the case of the spinless fermions) but of pointlike neutral complexes containing $2n$ particles and having a chirality $\pm 2n$.

For the ordinary electron system, $n = 2$, one has the complexes consisting of two right electrons and two left holes with opposite spins. For the conducting nanotubes $n = 4$ (Refs.5,6,7), and the complexes consists of eight particles. The complexes with a smaller number of particles can have a nonzero spin but their correlation is much weaker. For example, for $n = 2$ the spin phase can be realized only as a Kosterlitz-Thouless one, and for the limit of infinitely strong interaction the density tends to zero as $1/\sqrt{L_\parallel}$. By contrast, the spinless phase has for this limit a finite density.

This situation is typical for many field-theoretical models with Adler anomaly (this is the case for Luttinger model as well). It is known that in such models the new fermion interaction ("t'Hooft interaction"[8]) can appear with a vortex which is a product over all components of fermions. In many cases t'Hooft interaction leads to the spontaneous breakdown of chiral invariance with 2n fermion order parameter. In particular, this is a case for multicomponent Schwinger model[9] and, as we shall see, for the Luttinger model in the limit of infinitely strong interaction.

For last model the most correlated state is built out of complexes each of them having the maximal possible number of degrees of freedom. This state differs qualitatively from the result of a common treatment, i. e. marginal Wigner crystal ([3]). Instead of the phase transition of almost first kind one gets the phase transition of almost second kind. In order to manifest breakdown of the chiral symmetry in the Luttinger liquid we have calculated exactly the wave function of the ground state in this model, Eq. (27), and demonstrated explicitly that its symmetry is less than the original symmetry of the Hamiltonian. (This is the definition of spontaneous symmetry breaking). These are basically the main results of the paper.

For one component fermion system pointlike complexes with more than 2 particles are forbidden by Pauli principle. This is not the case for the multicomponent fermion. For this reason, expression for the ground state wave function Eq. (27) is much more cumbersome and the calculations are more involved. Therefore we will present the results only for a short range potential in the limit of infinitely strong interaction. At the same time, in our case the physical picture is quite similar to that of the one component fermions. For instance, had one taken into account corrections in the reciprocal strength of interaction one would have gotten a Kosterlitz-Thouless phase for spinless complexes too. The strength of the interaction going up, it would be gradually transformed into a state of a definite condensate density.

The paper is organized as follows. In Section I we have given a brief review of our results. Section II contains discussion of the main difference between the multicomponent problem and spinless one in regard of the theoretical description and most essential steps of calculation. In Section III we give arguments concerning the possibility to apply the theory to nanotubes. Appendix is devoted to a derivation of some intermediate results.

II. DESCRIPTION OF APPROACH AND DERIVATION OF THE MAIN RESULTS

Our starting point will be the usual Tomonaga — Luttinger Hamiltonian H (see, for instance, Refs.10, 2) for a system of interacting electrons without regard of the backscattering. For the case where the interaction does not change the electron spin the Hamiltonian H can be expressed through the density of the right (R) and left (L) electrons

$$\varrho\left(x\right)_{\alpha}=\varrho_{R,\alpha}\left(x\right)+\varrho_{L,\alpha}\left(x\right)$$

(the spin index α equals $\pm$ for the spin $\pm 1/2$ respectively) as

$$H=\sum_{\alpha}\int dx\left[\hat{\Psi}_{R,\alpha}^{\dagger}\left(x\right)v_{F}\left(-i\partial_{x}\right)\hat{\Psi}_{R,\alpha}\left(x\right)+\hat{\Psi}_{L,\alpha}^{\dagger}\left(x\right)v_{F}i\partial_{x}\hat{\Psi}_{L,\alpha}\left(x\right)\right]$$

$$+\int dxdy\varrho\left(x\right)V\left(x-y\right)\varrho\left(y\right). \tag{1}$$

Here v_F is the Fermi velocity and

$$\hat{\Psi}_{\alpha}\left(x\right)=\exp\left(ip_{F}x\right)\hat{\Psi}_{R,\alpha}\left(x\right)+\exp\left(-ip_{F}x\right)\hat{\Psi}_{L,\alpha}\left(x\right), \tag{2}$$

while ϱ is the total density. For simplicity, we assume that the interaction is spin-independent as well (it is not too essential for calculations). Now we introduce the electron and hole operators in the usual way

$$\hat{\Psi}_{(R,L)\alpha}\left(x\right)=\int_{0}^{\infty}\frac{dp}{2\pi}\left(\exp\left(\pm ipx\right)\hat{a}_{(R,L)\alpha}\left(p\right)+\exp\left(\mp ipx\right)\hat{b}_{(R,L)\alpha}^{\dagger}\left(p\right)\right)= \tag{3}$$

$$\hat{a}_{(R,L)\alpha}\left(x\right)+\hat{b}_{(R,L)\alpha}^{\dagger}\left(x\right)$$

Here $\hat{a}_{R,+}(x)$ is the operator of annihilation of an electron with spin $+1/2$ while $\hat{b}_{R,+}(x)$ is the operator of annihilation of a hole with spin $-1/2$. To write such interaction via functional integral it is necessary to introduce one Bose field Φ and apply a version of the Hubbard — Stratonovich identity[11]:

$$\exp\left[\frac{i}{2}\int_0^T dt\int_{-\infty}^{\infty}\frac{dp}{2\pi}V(p)\,\varrho(p,t)\,\varrho(-p,t)\right]=$$
$$\frac{1}{\mathcal{N}}\int\mathcal{D}\Phi\exp\left[\frac{i}{2}\int_0^T dt\int_{-\infty}^{\infty}\frac{dp}{2\pi}\Phi(p,t)\,\Phi(-p,t)\,V^{-1}(p)\right.$$
$$\left.-\frac{i}{2}\int_0^T dt\int_{-\infty}^{\infty}\frac{dp}{2\pi}\left(\varrho(p,t)\,\Phi(-p,t)+\varrho(-p,t)\,\Phi(p,t)\right)\right]. \tag{4}$$

Here $V(p)$ is the Fourier-transform of the e-e interaction and the normalization factor is equal to

$$\mathcal{N}=\int\mathcal{D}\Phi\exp\left[\frac{i}{2}\int_0^T dt\int_{-\infty}^{\infty}\frac{dp}{2\pi}\Phi(p,t)\,\Phi(-p,t)\,V^{-1}(p)\right]. \tag{5}$$

This identity shows that the theory with e-e interaction is equivalent to a theory of non-interacting electrons in an external field Φ. (Had one taken a theory with spin-dependent interaction he has to introduce two independent Bose fields. The calculation would be more cumbersome but the physical picture would be the same.)

Calculation of the ground state wave function is given in detail in Ref.[4]. It is based on calculation of the evolution operator for the electrons

$$S(T)=\sum_{m,n}|n><n|\exp(-iHT)|m><m|. \tag{6}$$

Here $|n>$ are the exact wave functions of the Hamiltonian H in the secondary quantization representation, T is the observation time. $S(T)$ determines the evolution of an arbitrary initial wave function ($<m|$) from the time $t=0$ up to final states $|n>$ (at $t=T$). (Henceforth we imply that the Schrödinger representation for operator with time-dependent wave-functions is used.)

Eq. (6) suggests the general method to obtain the wave functions. One has to calculate first the evolution operator and present it as a sum of time-dependent exponents. The coefficients in front of these exponents are products of the exact wave functions and their complex conjugates. In order to extract the ground state wave function one has to take the limit $T\to\infty$ (we add infinitesimal imaginary part to the energy). Proceeding to Euclidean time ($T\to -i/\Theta$) we see that evolution operator determines the density matrix for the equilibrium system at a non-zero temperature Θ.

As for the case of spinless electrons (see[4] for the details), one can present the evolution operator for the electrons in an external field as

$$\hat{S}(\Phi)=\exp(\mathcal{S}_0+\log[\,\mathrm{Det}\,\Phi(T)\,])|F><F|. \tag{7}$$

Before integration over the fields Φ the equation for $\hat{S}(\Phi)$ undergoes some changes in comparison with the spinless case. They amount to appearance of a factor n, the number of components of the electron wave function, in the equation describing the quantum fluctuations in the electron system under the action of the field $\Phi(T)$ (in the spin case $n=2$)

$$\ln\left[\mathrm{Det}\,\Phi(T)\right]=-\frac{n}{4\pi}\int_0^T dtdt_1\int_{-\infty}^{\infty}\frac{dp}{2\pi}\Phi(-p,t)\,\Phi(p,t_1)\,|p|\exp\left[-i|p|v_F|t-t_1|\right]. \tag{8}$$

The operator structure of Eq. (7) is determined by the second part of action, $\mathcal{S}_0$. Here one should take into account the spin index

$$\mathcal{S}_0=\sum_{i=R,L;\alpha}\int dxdx'\left[\hat{b}_{i,\alpha}(x')\,G_i^0(x',0;x,\epsilon)\,\hat{a}_{i,\alpha}(x)+\hat{a}_{i,\alpha}^{\dagger}(x')\,G_i^0(x',T;x,T-\epsilon)\,\hat{b}_{i,\alpha}^{\dagger}(x)\right.$$
$$\left.-\hat{a}_{i,\alpha}^{\dagger}(x')\,G_i^0(x',T;x,0)\,\hat{a}_{i,\alpha}(x)-\hat{b}_{i,\alpha}(x')\,G_i^0(x',0;x,T)\,\hat{b}_{i,\alpha}^{\dagger}(x)\right], \tag{9}$$

where G_i^0 is the spin-independent free particle Green function

$$G_{R,L}^0\left(x,t;x_1,t_1\right)=\frac{1}{2\pi i}\left[v_f\left(t-t_1\right)\mp\left(x-x_1\right)-i\delta\,\mathrm{sign}\left(t-t_1\right)\right]^{-1}. \tag{10}$$

Behaviour of the multicomponent fermions in the external field is quite similar to the one-component case. Essential complication, however, appears after the integration of the operator $\hat{S}\left(\Phi\right)$ over Φ with the weight (4).

Two points should be indicated.

- The coefficient n in Eq. (8) that enters the equation for the action. Due to this coefficient, after calculation of the integral over Φ the analytical structure of the resulting expression becomes much more complicated. As a result, in the integrals defining wave functions of the multiparticle complexes, one gets cuts instead of simple poles, as in one component case. This leads to rather cumbersome complex wave functions. In particular, complexes are non-local.

- The nonlocal property brings about essential enhancement of the various electron states. A number of electron states is forbidden for one component electrons due to Pauli principle. In contrast to this, in the multicomponent case the number of connected diagrams becomes infinite. This makes the expression for the ground state wave function rather cumbersome. However the very fact of existence of chiral phase for the infinitely strong interaction still persists.

Because of non-Gaussian form of the final functional integral it is impossible to perform the integration in Eq. (8) over $\Phi(x,t)$ in a closed form but it is possible to obtain an arbitrary term of the evolution operator expanding it in $\mathcal{S}_0^n$. It will be sufficient in order to get the ground state wave function as we will have the integral of a Gaussian type that can be easily calculated. After doing the final integration over Φ the final recipe of calculation of the evolution operator can be written as a sum of the following terms

$$\mathcal{S}_0^n\left(\hat{a}_{(R,L)},\hat{b}_{(R,L)},....\right)\exp\left(\mathcal{S}_n^{\mathrm{eff}}\right)|F><F|. \tag{11}$$

Here $|F>$ is the Fermi "sphere" while the terms in $\mathcal{S}_0^n$ determine the operator structure of the wave functions, i. e. all the possible particle configurations as a result of their interaction. (11) is a sort of symbolic expression. Indeed, the analytic equation for the effective action $\mathcal{S}_n^{\mathrm{eff}}$ in the nth term of expansion depends explicitly on the particle configuration in the preexponential factor $\left(\mathcal{S}_0\right)^n$. Naturally, it is different for different terms. Note that the evolution operator (6) is determined in such a manner that the initial state expressed through the electron and hole annihilation operators and the final state determined through creation operators are given at different times. It means that *during calculating the evolution operator* one should consider the operators $\hat{a}_{R,L}^{\dagger}\left(x\right)$ and $\hat{a}_{R,L}\left(y\right)$ as anticommuting.

In order to write the expression for $\mathcal{S}_n^{\mathrm{eff}}$, we introduce the following notation for the coordinates of the electron-hole creation-annihilation operators:

1. We will denote by x (y) the coordinates of the right (left) particles.

2. We will put a tilde on the coordinates related to annihilation operators (the initial state): the coordinates of creation operators (the final state) will have no tilde.

3. We will prime the hole coordinates.

The effective action differs from the action for the one-component fermions only by a factor and in the limit of strong interaction

$$\frac{V\left(p\right)}{\pi v_F}\gg 1$$

is equal

$$\mathcal{S}_{\mathrm{eff}}=-\frac{\pi}{nL}\sum_{m\neq 0}\frac{1}{|p_m|}\left[\mathcal{R}_f\left(-p,x_1\ldots\right)\mathcal{R}_f\left(p,x_1\ldots\right)+\mathcal{R}_i\left(-p,\tilde{x}_1\ldots\right)\mathcal{R}_i\left(p,\tilde{x}_1\ldots\right)\right]$$

$$-\frac{2\pi}{nL}\sum_{m\neq 0}\frac{1}{|p_m|}\exp\left(\frac{-|p_m|v_f}{\Theta}\right)\mathcal{R}_f\left(-p,x_1\ldots\right)\mathcal{R}_i\left(p,\tilde{x}_1\ldots\right), \tag{12}$$

The extra factor n is the number of the fermion components. It appears because in the Luttinger model the excitation spectrum is[12]:

$$\omega_p = |p|v_F\sqrt{1+\frac{nV\left(p\right)}{\pi v_F}}. \tag{13}$$

Eq. (12) is valid in the temperature region

$$\Theta_{\text{chiral}} \ll \Theta \ll \Theta_c = \omega_{(p_{\min})}.$$

The right-hand side of the last inequality is the energy of excitations with a minimal momentum (for periodic boundary conditions $p_{\min} = 2\pi/L_{\|}$) while $\Theta_{\text{chiral}} = |p_{\min}|v_F$ is the degeneracy temperature of the ground state. In this temperature region one can neglect the energy difference between the states of different chiralities. This means that the ground states with different chiralities becomes degenerate.

The origin of this inequality has been discussed in detail in Ref.[4]. It is not sensitive to the number of the wave function components. For the temperatures $\Theta \ll \Theta_{\text{chiral}}$ the last term in Eq. (12) should be omitted. Then the corresponding equation will be valid for the low temperatures too (see detailed discussion in Ref. [4]).

The functions $\mathcal{R}_{i,f}\left(p,\tilde{x}_1\ldots\right)$ in the equation (12) depend explicitly on the electron (x..) and hole ($\tilde{x}$...) coordinates in the preexponential factor. These functions are given by

$$\begin{aligned}\mathcal{R}_f\left(p,x\ldots\right) &= \sum_{x\ldots;x'\ldots;y\ldots;y'\ldots}\theta\left(p\right)\left[\exp\left(ipx\right)\right.\\ &\left.-\exp\left(ipx'\right)\right]+\theta\left(-p\right)\left[\exp\left(ipy\right)-\exp\left(ipy'\right)\right],\\ \mathcal{R}_i\left(p,\tilde{x}...\right) &= \sum_{\tilde{x}\ldots;\tilde{x}'\ldots;\tilde{y}\ldots;\tilde{y}'\ldots}\theta\left(-p\right)\left[\exp\left(ip\tilde{x}\right)\right.\\ &\left.-\exp\left(ip\tilde{x}'\right)\right]+\theta\left(p\right)\left[\exp\left(ip\tilde{y}\right)-\exp\left(ip\tilde{y}'\right)\right]\end{aligned} \tag{14}$$

To obtain the complete expression for the ground state wave function one has to consider all the complexes and separate the connected parts out of them. It is not necessary, however, since, according to the general theorem, the complete wave function is the exponent of the connected complexes[13].

For one-component fermions there is only one possible 2-particle connected complex. This is not true for multi-component case, many of the scattering channels are possible, so the number of connected diagrams is infinite. In principle, one can calculate the exact wave function of any given complex taking the Gaussian integral in Φ. Unfortunately, it is not enough to present the whole wave function of the system in a closed form, but actually in order to prove the existence of symmetry breaking we do not needed it. To be sure of the fact it is sufficient to prove that the wave function symmetry is less than Hamiltonian. In order to do this one should analyze the simplest connected diagrams bringing about a spontaneous breaking of the Hamiltonian symmetry. The rest terms either have the symmetry of the Hamiltonian or describe the scattering of the simplest correlated complexes and also violate the chiral symmetry.

Now we embark on analysis of the simplest diagrams of the evolution operator for the electrons having a spin. We will begin from the temperature region $\Theta_{\text{chiral}} \gg \Theta$. In this case the action and therefore the evolution operator factorize so that one can consider explicitly the ground state wave function $|\Omega>$. The simplest nontrivial diagram we are interested in is

$$\begin{aligned}\int\frac{dx_+dx'_+dx_-dx'_-}{\left(2\pi i\right)^2}\frac{dy_+dy'_+dy_-dy'_-}{\left(2\pi i\right)^2}\frac{\hat{a}^\dagger_{R,+}\left(x_+\right)\hat{b}^\dagger_{R,+}\left(x'_+\right)}{x'_+-x_+-i\delta}\frac{\hat{a}^\dagger_{R,-}\left(x_-\right)\hat{b}^\dagger_{R,-}\left(x'_-\right)}{x'_--x_--i\delta}\\ \frac{\hat{a}^\dagger_{L,+}\left(y_+\right)\hat{b}^\dagger_{L,+}\left(y'_+\right)}{y_+-y'_+-i\delta}\frac{\hat{a}^\dagger_{L,-}\left(y_-\right)\hat{b}^\dagger_{L,-}\left(y'_-\right)}{y_--y'_--i\delta}\exp[\mathcal{S}^f_{\text{eff}}\left(x_+,\ldots\right)]|F>.\end{aligned} \tag{15}$$

Further on we will see that the terms with a smaller number of operators give a weaker correlation than Eq. (15).

The action for this electrons-holes configuration is

$$\mathcal{S}^f_{\text{eff}}\left(x,_\alpha,\ldots\right)=\frac{1}{2}\ln\frac{\prod_{\alpha,\alpha'\ldots}\left(x_\alpha-y_\beta+i\delta\right)\left(x'_{\alpha'}-y'_{\beta'}+i\delta\right)}{\prod_{\alpha,\alpha'\ldots}\left(x'_\alpha-y_\beta+i\delta\right)\left(x_{\alpha'}-y'_{\beta'}+i\delta\right)}. \tag{16}$$

In fact, it differs from the corresponding expression for one-component fermions by the factor 1/2. This factor leads to the singularities of the integrand Eq. (15) which are cuts instead of simple poles in one component case. This prevents us from explicit calculation of the integral.

Nevertheless, it is possible to recognize the spontaneous breakdown of the chiral symmetry in our system. To do this, several steps are necessary. First, we have to analyze what new bound complexes appeared as a result of interaction. We have to take the arbitrary connected diagram and try to separate complexes with a smaller number of particles out of it. To do this one has to consider all particles in one complex as being one close to another, while the distances between different complexes are large. If the full wave function in this limit decays into the product of two wave functions, one depending only on the coordinates of first complex, the other only on coordinates of the second one, then the complexes can be considered as new "free particles"[14], as the probability to find one such complex does not depend on the position of another one. In other words, we should present a term of expansion of evolution operator we consider as a product of the form:

$$\int\frac{dx_+dy_+}{2\pi i}\ldots\hat{a}^\dagger_{R,+}\left(x_+\right)\ldots K\left(x_+,\ldots,y_+\ldots;\right)a^\dagger_{L,+}\left(y_+\right)\ldots.$$

Then we should check that provided the complex $(x_+,\ldots,)$ is moved off from the complex $(y_+\ldots;)$ over the distance of the order of $L_\parallel$ the amplitude $K\left(x_+,\ldots,y_+,\ldots\right)$ tends not to zero (as is usually the case with scattering amplitudes) but to the factorized product $k\left(x_+,\ldots,\right)k_1\left(y_+,\ldots\right)$. Here each factor depends on the variables of the first or second group. This means that two complexes are formed as a result of the interaction. If the intercomplex distance is large enough, their contribution to the wave function can be presented as follows

$$\int\frac{dx_+}{\sqrt{2\pi i}}\ldots k\left(x_+\ldots,\right)\hat{a}^\dagger_{R,+}\left(x_+\ldots\right)\int\frac{dy_+}{\sqrt{2\pi i}}k_1\left(y_+\ldots,\right)\hat{a}^\dagger_{L,+}\left(y_+\ldots\right)|F>.$$

The remnant part of K (which is $K-k\cdot k_1$) is a connected diagram which describes the intercomplex scattering. The theorem of logarithm connectedness[13] guarantees that the same connected complexes will appear in the next orders as well with correct combinatorial coefficients and the final answer is an exponent of connected complexes. In particular, the first order term

$$\int\frac{dx_+}{\sqrt{2\pi i}}\ldots k\left(x_+,\ldots\right)\hat{a}^\dagger_{R,+}\left(x_+\right)\ldots|F>$$

should appear in the expansion of evolution operator directly, unless it is forbidden by some conservation law (e.g. chirality conservation for the lowest temperature case in our model). In this case one has to use the projector on the proper state as in (Eq.24)).

Whether a phase transition of the second kind exists or not, depends on the symmetry of the complexes $\hat{a}^\dagger_{R,+}\left(x_+\right)\ldots$ If they are less symmetric than the initial Hamiltonian one has a symmetry breaking. As a result it is possible to introduce a non-vanishing order parameter in the less symmetric phase while it vanishes in the more symmetric one where the complexes do not exist. (More precisely, taking into account a fluctuations of the low symmetry phase in phase with non-broken symmetry one can state that an order parameter should not enhance with $L_\parallel$ in the last case. Our definition the order parameter is given in Eq.(29).) According to Landau (see Landau and Lifshitz[15]) this is the definition of the phase transition of the second kind. If, however, the symmetries of all the connected complexes and of the Hamiltonian are the same one has a long-range correlations without spontaneous symmetry breaking.

Now we embark on analysis of the simplest diagrams of the evolution operator. By analogy with the theory of one component fermions one could assume that the simplest connected diagrams originate from the term

$$\int\frac{dx_+dx'_+}{2\pi i}\frac{dy_-dy'_-}{2\pi i}\frac{\hat{a}^\dagger_{R,+}\left(x_+\right)\hat{b}^\dagger_{R,+}\left(x'_+\right)}{x'_+-x_+-i\delta}\frac{\hat{a}^\dagger_{L,-}\left(y_-\right)\hat{b}^\dagger_{L,-}\left(y'_-\right)}{y_--y'_--i\delta}\exp[\mathcal{S}^f_{\text{eff}}\left(x_+,\ldots\right)]|F>. \tag{17}$$

However, because of the factor n^{-1} the corresponding contribution to the action is

$$\mathcal{S}_{\text{eff}}^{f}\left(x_{+},\ldots\right)=\frac{1}{2}\ln\frac{\left(x_{+}-y_{-}+i\delta\right)\left(x_{+}^{\prime}-y_{-}^{\prime}+i\delta\right)}{\left(x_{+}^{\prime}-y_{-}+i\delta\right)\left(x_{+}-y_{-}^{\prime}+i\delta\right)}. \tag{18}$$

The bound chiral complexes are determined by the singularities of the integrand at $|x_{+}^{\prime}-y_{-}|\sim d,|x_{+}-y_{-}^{\prime}|\sim d,|x_{+}-y_{-}|\sim L_{\|}$ (here d is the width of the conductor). As a result, the contribution we are interested in is of the order

$$\int dx_{+}dy_{-}\hat{a}_{R}^{\dagger}\left(x_{+}\right)\hat{b}_{L}^{\dagger}\left(x_{+}\right)\hat{a}_{L}^{\dagger}\left(y_{-}\right)\hat{b}_{R}^{\dagger}\left(y_{-}\right)\frac{d}{|x_{+}-y_{-}|}|F>.$$

This quantity tends to 0 at $|x_{+}-y_{-}|\rightarrow\infty$, but more slowly than for free particle[16] case.

Now we will show that the most correlated state can be obtained from Eq.(15). It can be split into two four-particle complexes, each with zero spin, having the chiral charges ±4 ($\hat{a}_{R,+}^{\dagger}\hat{a}_{R,-}^{\dagger}\hat{b}_{L,+}^{\dagger}\hat{b}_{L,-}^{\dagger}$ and $\hat{a}_{L,+}^{\dagger}\hat{a}_{L,-}^{\dagger}\hat{b}_{R,+}^{\dagger}\hat{b}_{R,-}^{\dagger}$). The amplitude K in Eq.(15) factorizes and does not tend to 0 at $L_{\|}\rightarrow\infty$. Indeed, the c-factor in the integrand is

$$\begin{aligned}K\left(x_{+},\ldots\right)=\frac{1}{x_{+}^{\prime}-x_{+}-i\delta}\frac{1}{x_{-}^{\prime}-x_{-}-i\delta}\frac{1}{y_{+}-y_{+}^{\prime}-i\delta}\frac{1}{y_{-}-y_{-}^{\prime}-i\delta}\\ \frac{\sqrt{\prod_{\alpha,\alpha^{\prime}\ldots}\left(x_{\alpha}-y_{\beta}+i\delta\right)\left(x_{\alpha^{\prime}}^{\prime}-y_{\beta^{\prime}}^{\prime}+i\delta\right)}}{\sqrt{\prod_{\alpha,\alpha^{\prime}\ldots}\left(x_{\alpha}^{\prime}-y_{\beta}+i\delta\right)\left(x_{\alpha^{\prime}}-y_{\beta^{\prime}}^{\prime}+i\delta\right)}}.\end{aligned} \tag{19}$$

Now, let us consider the regions of integration $x_{+}\sim x_{-}\sim y_{-}^{\prime}\sim y_{+}^{\prime}$ and $x_{+}^{\prime}\sim x_{-}^{\prime}\sim y_{+}\sim y_{-}$ assuming that the distances between these groups of variables are of the order of $L_{\|}$. Then the amplitude K tends to

$$\begin{aligned}V_{+4}\left(x_{+},\ldots\right)V_{-4}\left(x_{+}^{\prime},\ldots\right)=\\ =1/\sqrt{\left(x_{+}-y_{+}^{\prime}+i\delta\right)\left(x_{+}-y_{-}^{\prime}+i\delta\right)\left(x_{-}-y_{+}^{\prime}+i\delta\right)\left(x_{-}-y_{-}^{\prime}+i\delta\right)}\\ 1/\sqrt{\left(x_{+}^{\prime}-y_{+}+i\delta\right)\left(x_{+}^{\prime}-y_{-}+i\delta\right)\left(x_{-}^{\prime}-y_{+}+i\delta\right)\left(x_{-}^{\prime}-y_{-}+i\delta\right)},\end{aligned} \tag{20}$$

This means that each amplitude V depends on the variables belonging either to the first or to the second group. This property of the amplitude permits one to single out of the full equation for the evolution operator the connected complexes and the amplitude of intercomplex scattering that tend to 0 for large intercomplex distances.

This term of expansion, besides the chiral complexes, has also neutral ones with zero chirality, $\hat{a}_{R,+}^{\dagger}\hat{b}_{R,+}^{\dagger}\hat{a}_{L,-}^{\dagger}\hat{b}_{L,-}^{\dagger}$ and $\hat{a}_{R,-}^{\dagger}\hat{b}_{R,-}^{\dagger}\hat{a}_{L,+}^{\dagger}\hat{b}_{L,+}^{\dagger}$. They do not violate the symmetry of the Hamiltonian. However, they should be singled out, so that the remaining scattering amplitude tended to zero in the whole region of the variable variation. This permits one to interpret it as the intercomplex scattering amplitude. The zero chirality complexes are not important for existence of the phase transition. However, one should take them into account for calculation of the matrix elements as the theory has no small parameter to neglect them. To check that they exist we consider in (15) the region $x_{+}\sim x_{+}^{\prime}\sim y_{-}\sim y_{-}^{\prime}$ $x_{-}\sim x_{-}^{\prime}\sim y_{+}\sim y_{+}^{\prime}$. In this region the amplitude K tends to $V_{0}\left(x_{+},\ldots\right)V_{0}\left(x_{-},\ldots\right)$ where

$$V_{0}\left(x_{\alpha},\ldots\right)=\frac{1}{\left(x_{\alpha}^{\prime}-x_{\alpha}-i\delta\right)\left(y_{-\alpha}-y_{-\alpha}^{\prime}-i\delta\right)}\frac{\sqrt{\left(x_{\alpha}-y_{-\alpha}+i\delta\right)\left(x_{\alpha}^{\prime}-y_{-\alpha}^{\prime}+i\delta\right)}}{\sqrt{\left(x_{\alpha}-y_{-\alpha}^{\prime}+i\delta\right)\left(x_{\alpha}^{\prime}-y_{-\alpha}^{\prime}+i\delta\right)}}. \tag{21}$$

This means that this quantity can be presented as a product of the amplitudes, each of them remaining finite provided the distance between them tends to infinity.

Now it is convenient to introduce the amplitude of intercomplex scattering V_{coll}. By derivation, it tends to zero, the intercomplex distance tending to ∞:

$$V_{coll}\left(x_{+},\ldots\right)=K\left(x_{+},\ldots\right)-V_{+4}\left(x_{+},\ldots\right)V_{-4}\left(x_{+}^{\prime},\ldots\right)-V_{0}\left(x_{+},\ldots\right)V_{0}\left(x_{-},\ldots\right). \tag{22}$$

The contribution to the ground state wave function we are interested in can be presented through these amplitudes as

$$\int \frac{dx_+dx'_+dx_-dx'_-}{(2\pi i)^2}\frac{dy_+dy'_+dy_-dy'_-}{(2\pi i)^2}\hat{a}^\dagger_{R,+}(x_+)\,\hat{b}^\dagger_{R,+}(x'_+)\,\hat{a}^\dagger_{R,-}(x_-)$$
$$\hat{b}^\dagger_{R,-}(x'_-)\,\hat{a}^\dagger_{L,+}(y_+)\,\hat{b}^\dagger_{L,+}(y'_+)\,\hat{a}^\dagger_{L,-}(y_-)\,\hat{b}^\dagger_{L,-}(y'_-)$$
$$\left(V_{+4}(x_+,\ldots)\,V_{-4}(x'_+,\ldots)+V_0(x_+,\ldots)\,V_0(x_-,\ldots)+V_{coll}(x_+,\ldots)\right)|F>. \quad (23)$$

First term here describes the non-interacting complexes with non-zero chirality, second describes the 4-particle neutral complexes and third (connected part) is related to their interaction. We are mainly interested in first term as it is connected with breakdown of chiral symmetry.

The chiral complex which we obtained is already a connected one and cannot be separated to simpler ones. This means that its wave function is a decreasing function of interparticle distances. It is shown in the Appendix that probability to find particles of the complex far from each other is negligibly small.

Besides, one should take into consideration that in the temperature interval $\Theta \ll \Theta_{\rm chiral}$ where one need not consider the last term in Eq. (12) there is one to one correspondence between the complexes with chirality $Q=4$ and $Q=-4$, so that the total chirality of the state equals zero. The theorem of logarithm connectedness states that the ground state wave function can be presented as

$$|\Omega>=P_{C=0}\exp\sum_\alpha \mathrm{Tr}\left[\frac{1}{(2\pi i)^2}V_{+4}(x_\alpha,x_{-\alpha},y'_{-\alpha},y'_\alpha)\left(\hat{a}^\dagger_{R,\alpha}(x_\alpha)\,\hat{a}^\dagger_{R,-\alpha}(x_{-\alpha})\right.\right.$$
$$\times\,\hat{b}^\dagger_{L,-\alpha}(y'_{-\alpha})\,\hat{b}^\dagger_{L,\alpha}(y'_\alpha)+\hat{a}^\dagger_{L,\alpha}(y'_\alpha)\,\hat{a}^\dagger_{L,-\alpha}(y'_{-\alpha})\,\hat{b}^\dagger_{R,-\alpha}(x_{-\alpha})\,\hat{b}^\dagger_{R,_\alpha}(x_\alpha)\Big)$$
$$+\frac{1}{(2\pi i)^2}V_{+0}(x_\alpha,x_{-\alpha},y'_{-\alpha},y'_\alpha)\left(\hat{a}^\dagger_{R,\alpha}(x_\alpha)\,\hat{b}^\dagger_{R,\alpha}(x'_\alpha)\right.$$
$$\times\,\hat{a}^\dagger_{L,-\alpha}(y_{-\alpha})\,\hat{b}^\dagger_{L,-\alpha}(y'_{-\alpha})+\hat{a}^\dagger_{R,-\alpha}(x_{-}\alpha)\,\hat{b}^\dagger_{R,-\alpha}(x'_{-\alpha})\,\hat{a}^\dagger_{L,\alpha}(y_\alpha)\,\hat{b}^\dagger_{L,\alpha}(y'_\alpha)\Big)$$
$$+\frac{1}{(2\pi i)^4}V_{coll}(x_\alpha,x_{-\alpha},...)\,\hat{a}^\dagger_{R,\alpha}(x_\alpha)\,\hat{a}^\dagger_{R,-\alpha}(x_{-\alpha})\,\hat{b}^\dagger_{L,-\alpha}(y'_{-\alpha})\,\hat{b}^\dagger_{L,\alpha}(y'_\alpha)$$
$$\left.\times\,\hat{a}^\dagger_{L,\alpha}(y_\alpha)\,\hat{a}^\dagger_{L,-\alpha}(y_{-\alpha})\,\hat{b}^\dagger_{R,-\alpha}(x'_{-\alpha})\,\hat{b}^\dagger_{R,_\alpha}(x'_\alpha)+\ldots\right]. \quad (24)$$

Here $P_{C=0}$ is the projector on the state with zero chirality. The symbol Tr includes the integrations in over particle coordinates. The terms omitted in Eq. (24) describe scattering of three and more complexes while all the elementary complexes are present here. Mark that the complexes with non-zero chirality have appeared in the theory. Nevertheless, the wave function of the ground state as a whole described the state with $Q=0$ in this temperature region, i. e. the symmetry of the ground state is the same as that of the Hamiltonian. The states with nonzero chirality have a bigger energy (of the order of $2\pi v_F/L_\parallel$). Therefore the spontaneous symmetry breaking may take place only in the region of higher temperatures $\Theta \gg \Theta_{\rm chiral}$ where such an energy difference is not essential. In this temperature region one should consider in the equation (9) for the action $\mathcal{S}_0$ also the term where the time arguments of the Green functions differ by T. (Practically it is more convenient to introduce the temperature by replacement $T\to -i/\Theta$ in the final equations.) Then one has the following non-trivial term in the operator of evolution

$$\mathrm{Tr}\frac{1}{(2\pi i)^4}\frac{\hat{a}^\dagger_{R,+}(x_+)\,\hat{a}_R(\tilde{x}_+)}{\tilde{x}_+-x_++v_fT-i\delta}\frac{\hat{a}^\dagger_{R,-}(x_-)\,\hat{a}_R(\tilde{x}_-)}{\tilde{x}_--x_-+v_fT-i\delta}$$
$$\frac{\hat{b}^\dagger_{L,+}(y'_+)\,\hat{b}_L(\tilde{y}'_+)}{\tilde{y}'_+-y'_++v_fT+i\delta}\frac{\hat{b}^\dagger_{L,-}(y'_-)\,\hat{b}_L(\tilde{y}'_-)}{\tilde{y}'_--y'_--v_fT+i\delta}\exp[\mathcal{S}^f_{\rm eff}](x_+,\ldots)|F><F|. \quad (25)$$

The action $\mathcal{S}^f_{\rm eff}$ for this configuration is

$$\frac{1}{2}\ln\frac{\prod_{\alpha,\alpha'\ldots}\left(\tilde{y}'_\alpha-y'_\beta-v_fT+i\delta\right)(x_{\alpha'}-\tilde{x}_{\beta'}-v_fT+i\delta)}{\prod_{\alpha,\alpha'\ldots}\left(x_\alpha-y'_\beta+i\delta\right)(\tilde{y}'_{\alpha'}-\tilde{x}_{\beta'}+i\delta)}. \quad (26)$$

It is readily seen out of the operator structure of this term that the amplitude V_4 appears here automatically (without extracting the amplitudes of the neutral complexes and channels of scattering). It is a consequence of the theorem of logarithm connectedness. It guarantees coincidence of the amplitude in this term with V_4. Indeed, in the region where the same variables with spin up and spin down are quite near to each other (for instance, $x_\alpha \sim x_{-\alpha}$) while the coordinates in the creation and annihilation operators are apart at the distance of the order of $L_\parallel$ ($x_\alpha \sim \tilde{x}_\alpha \sim L \gg 2\pi v_f/\Theta$) all the c-factor in the integrand of (25) turns into the factorized expression

$$V_{+4}\left(x_+,\ldots\right)V_{-4}^*\left(\tilde{x}_+,\ldots\right).$$

This proves that the chiral four particle complexes were singled properly out of a more complicated expression (22). Such terms in the evolution operator result in any chirality of the ground state. However, any state with a fixed chirality should be unstable relative to the backscattering, however weak, violating the chirality. Therefore one should consider a superposition of all such states. As a result, one has, in the same way as in the theory of superconductivity, to introduce a condensate with a fixed phase instead of a state with a fixed chirality:

$$\begin{aligned}
|\Omega>_\theta = \exp\sum_\alpha \mathrm{Tr}\Bigg[&\frac{1}{(2\pi i)^2}V_{+4}\left(x_\alpha, x_{-\alpha}, y'_{-\alpha}, y'_\alpha\right)\Big(\exp\left(i\theta\right)\hat{a}^\dagger_{R,\alpha}\left(x_\alpha\right)\hat{a}^\dagger_{R,-\alpha}\left(x_{-\alpha}\right)\\
\times\ &\hat{b}^\dagger_{L,-\alpha}\left(y'_{-\alpha}\right)\hat{b}^\dagger_{L,\alpha}\left(y'_\alpha\right)+\exp\left(-i\theta\right)\hat{a}^\dagger_{L,\alpha}\left(y'_\alpha\right)\hat{a}^\dagger_{L,-\alpha}\left(y'_{-\alpha}\right)\hat{b}^\dagger_{R,-\alpha}\left(x_{-\alpha}\right)\hat{b}^\dagger_{R,_\alpha}\left(x_\alpha\right)\Big)\\
&+\frac{1}{(2\pi i)^2}V_{+0}\left(x_\alpha, x_{-\alpha}, y'_{-\alpha}, y'_\alpha\right)\Big(\hat{a}^\dagger_{R,\alpha}\left(x_\alpha\right)\hat{b}^\dagger_{R,\alpha}\left(x'_\alpha\right)\\
\times\ &\hat{a}^\dagger_{L,-\alpha}\left(y_{-\alpha}\right)\hat{b}^\dagger_{L,-\alpha}\left(y'_{-\alpha}\right)+\hat{a}^\dagger_{R,-\alpha}\left(x_{-}\alpha\right)\hat{b}^\dagger_{R,-\alpha}\left(x'_{-\alpha}\right)\hat{a}^\dagger_{L,\alpha}\left(y_\alpha\right)\hat{b}^\dagger_{L,\alpha}\left(y'_\alpha\right)\Big)\\
&+\frac{1}{(2\pi i)^4}V_{coll}\left(x_\alpha, x_{-\alpha}, ...\right)\hat{a}^\dagger_{R,\alpha}\left(x_\alpha\right)\hat{a}^\dagger_{R,-\alpha}\left(x_{-\alpha}\right)\hat{b}^\dagger_{L,-\alpha}\left(y'_{-\alpha}\right)\hat{b}^\dagger_{L,\alpha}\left(y'_\alpha\right)\\
&\times\ \hat{a}^\dagger_{L,\alpha}\left(y_\alpha\right)\hat{a}^\dagger_{L,-\alpha}\left(y_{-\alpha}\right)\hat{b}^\dagger_{R,-\alpha}\left(x'_{-\alpha}\right)\hat{b}^\dagger_{R,_\alpha}\left(x'_\alpha\right)+....\Bigg],
\end{aligned}\tag{27}$$

Eq. (27) demonstrates that for a strong electron-electron interaction a spontaneous symmetry breaking takes place. Two first terms in the equation for the ground state wave function are not invariant under the chiral transformation

$$\Psi_{R,L}\left(x\right)\rightarrow\exp\left(\pm i\Lambda\right)\Psi_{R,L},\tag{28}$$

while the Hamiltonian still retains the invariance. (Here Λ is a constant.)

Such a form of the bound state permits to introduce the order parameter. It equals to zero (or, to be more exact, not grows with $L_\parallel$) in the phase of high symmetry ($\Theta \gg \Theta_c = \omega_{p_{\min}}$) and is proportional to $L_\parallel$ in the phase of low symmetry for low temperatures. (One can prove the first statement using ordinary symmetry considerations or by direct analytical calculation.) For our case the following quantity can be considered as the order parameter

$$\int dx\ {}_\theta<\Omega|\hat{a}^\dagger_{R,\alpha}\left(x\right)\hat{a}^\dagger_{R,-\alpha}\left(x\right)\hat{b}^\dagger_{L,\alpha}\left(x\right)\hat{b}^\dagger_{L,-\alpha}\left(x\right)|\Omega>_\theta \sim L_\parallel.\tag{29}$$

One can see from Eq. (29) as to why the phase transition of the second kind demands the chirality degeneracy of the ground state. For the order parameter to be nonvanishing it is necessary to be able to add to the ground state an extra four particle complex. This demands condition $\Theta \gg \Theta_{\mathrm{chiral}}$ because zero chirality state has the lowest energy and the energy difference between the state and other ones about Θ_{chiral}. There is also the upper bound of existence of the low symmetry phase

$$\Theta \ll \omega_{p_{\min}}.\tag{30}$$

This limitation is due to the fact that in one dimensional systems the long range order is suppressed by the thermal excitations. In the temperature region given by Eq. (30) one can neglect it; this condition does

not depend on the number of the fermion components and is discussed in detail in[4]. So, the temperature region

$$\Theta_{\rm chiral} < \Theta < \Theta_c \tag{31}$$

is the region where the broken phase exists.

Thus in the system of interacting multicomponent fermions the most correlated state consists of $2n$ operators and has the chirality $\pm 2n$. It is this state that in the limit of infinitely strong interaction results in the phase transition of the second kind. At the same time the spin complexes of a smaller number of operators, violating the symmetry of the ground state, can exist only as a Kosterlitz — Thouless phase. For the limit of infinitely strong interaction their density tends to zero as $1/\sqrt{L_\parallel}$. By contrast, the spinless phase has for this case a finite density. This rule is quite general.

III. CARBON NANOTUBES

The conducting carbon nanotubes give one more example of multicomponent electrons. In order to generalize the theory developed above one should have one dimensional conducting tubes and such e-e interaction that could be rewritten in the density-density form — see Eq.(1). It means that one should be able to neglect a backward and inter-component (see below) scattering. Following Ref. 17 one can visualize a nanotube as a cylinder constructed of a monoatomic layer of graphite. The latter has a lattice of adjoining regular hexagons, so that the angle between the neighboring basis vectors, n_a and n_b is $2\pi/3$. Choosing the coordinates ξ_1 and ξ_2 in such a way that axis $0\xi_1$ is parallel to $\mathbf{a}$ while axis $0\xi_2$ is perpendicular to $\mathbf{a}$ one can present these vectors as

$$\mathbf{a} = a(1,0), \quad \mathbf{b} = a(-1/2, \sqrt{3}/2). \tag{32}$$

a is the lattice constant that is equal to $d\sqrt{3}$, d =1.44 Å being the interatomic distance[18].

The circumferential vector $\mathbf{L}$ can be written as

$$\mathbf{L} = n_a\mathbf{a} + n_b\mathbf{b}. \tag{33}$$

Here n_a and n_b are integers.

The electron effective Hamiltonian for a graphite sheet is

$$H = \begin{pmatrix} 0 & h^* \\ h & 0 \end{pmatrix}. \tag{34}$$

It can be expanded in the vicinity of the points

$$\mathbf{P} = (4\pi/3a)(-1,0), \quad \mathbf{P}' = (4\pi/3a)(1,0) \tag{35}$$

up to the first power in the small deviations $\mathbf{p}$ and $\mathbf{p}'$ from the values given by first and second Eq. (35) respectively

$$h(\mathbf{P},\mathbf{p}) = \gamma e^{-i\theta}\left(p_\perp - ip_z\right), \quad h(\mathbf{P}',\mathbf{p}') = \gamma e^{i\theta}\left(-p'_\perp - ip'_z\right). \tag{36}$$

Here $\gamma = (\sqrt{3}/2)\gamma_0 a$; $\gamma_0 \approx$3eV (see 17,18) is the transfer integral between the neighboring π orbitals while θ is the angle between vectors $\mathbf{L}$ and $\mathbf{a}$. The subscripts z and $\perp$ refer to the components of $\mathbf{p}$ relative to the direction of $\mathbf{L}$, namely $p_z \perp \mathbf{L}$ and $p_\perp \| \mathbf{L}$, so that $p_z(p_\perp)$ is parallel (perpendicular) to the axis of the tube.

The spectrum near the point $\mathbf{P}$ is given by

$$E(\mathbf{P},\mathbf{p}) = \pm\gamma\sqrt{p_z^2 + p_\perp^2} \tag{37}$$

where the upper (lower) sign corresponds to the conduction (valence) band in this equation. The spectrum near the $\mathbf{P}'$ point is obtained by the replacement $\mathbf{p} \to \mathbf{p}'$.

The electron wave function $\Psi(\mathbf{r})$ should satisfy the cyclic boundary condition

$$\Psi(\mathbf{r}) = \Psi(\mathbf{r} + \mathbf{L}), \tag{38}$$

so that the discrete values of $p_\perp$ and $p'_\perp$ are given by (see Ref. 17)

$$p_\perp = \frac{2\pi}{|\mathbf{L}|}\left(n - \frac{\nu}{3}\right); \quad p'_\perp = \frac{2\pi}{|\mathbf{L}|}\left(n + \frac{\nu}{3}\right). \tag{39}$$

Here $n = 0, \pm 1, \pm 2, \ldots$ while $|\mathbf{L}| = a\sqrt{n_a^2 + n_b^2 - n_a n_b}$; $\nu = 0$ or ± 1 and is determined by the presentation of the sum $n_a + n_b$ as $3N + \nu$ (N being an integer). The nanotubes are conductive (metallic) for the combination

$$n = \nu = 0 \tag{40}$$

and we will consider this case for our further analysis. In other words, in such tubes there are two conic bands, i. e. the points $\alpha_P \mathbf{P}$ with $\alpha_P = \pm 1$. The big phase corresponding to the momentum $\alpha_P \mathbf{P}$ should be extracted in the same way as it has been done in Eq. 2. Besides, we assume that due to the presence of gate electrodes the Fermi level is well above (or below) the points $\alpha_P \mathbf{P}$ (cf. with Ref.[6]). As a result, we will have a theory with four-component fermions. There are two extra branches corresponding to two values of α_P. In each of them there are analogues of R- and L-particles. To introduce them one should, in full analogy with Eq. (2), separate the corresponding phase factors with large phases. In order to establish correspondence between the present model and the Luttinger one we should be able to neglect both the transitions between different branches (different values of α_P) and within the same branch between R- and L-particles. As indicated in Ref. 19, the nanotubes have comparatively large radii that encompass with ($N \gg 1$) atoms. Therefore the only e-e scattering that is important in this limit is the forward one that involves a small quasimomentum transfer. The matrix element describing the backscattering within a band as well as the $\mathbf{P} \leftrightarrow \mathbf{P}'$ scattering acquire an extra small factor of the order of $1/N$. This is why one can neglect these types of scattering. This means that we can use the results obtained in Section II. Repeating the arguments of this section for $n = 4$ one can come to the conclusion that a condensate is formed in the ground state. It consists of the eights of the form

$$\hat{a}^\dagger_{R,\alpha,\alpha_P}(x)\,\hat{a}^\dagger_{R,-\alpha,\alpha_P}(x)\,\hat{b}^\dagger_{L,\alpha,\alpha_P}(x)\,\hat{b}^\dagger_{L,-\alpha,\alpha_P}(x)\,\hat{a}^\dagger_{R,\alpha,-\alpha_P}(x)\,\hat{a}^\dagger_{R,-\alpha,-\alpha_P}(x)\,\hat{b}^\dagger_{L,\alpha,-\alpha_P}(x)\,\hat{b}^\dagger_{L,-\alpha,-\alpha_P}(x),$$

Their chirality is ± 8.

Acknowledgments

V. V. A. and V. L. G. are grateful for partial support the work by the Russian National Foundation for Basic Research, grant No 06-02-16384.

APPENDIX A: CHARACTERISTIC DIMENSIONS OF CORRELATED COMPLEXES

The simplest way to give the proof of existence of a bound chiral complex and to determine its characteristic dimensions is to consider the state with a single chiral complex:

$$|\Phi_c> = \mathrm{Tr}\left[\frac{\hat{a}^\dagger_{R,+}(x_+)\hat{a}^\dagger_{R,-}(x_-)\hat{b}^\dagger_{L,-}(y'_-)\hat{b}^\dagger_{L,+}(y'_+)}{\sqrt{\prod_{\alpha,\alpha'=\pm}(x_\alpha - y'_{\alpha'} + i\delta)(x_\alpha - y'_{\alpha'} + i\delta)}}\right]|F>. \tag{A1}$$

For instance, one can calculate the probability to find an electron with spin up at the distance $|z_+ - z_-| > d$ from the electron with spin down as

$$<\Phi_c|\hat{a}^\dagger_{R,+}(z_+)\,a_{R,+}(z_+)\,\hat{a}^\dagger_{R,-}(z_-)\,\hat{a}_{R,-}(z_-)\,|\Phi_c>.$$

Moving all the creation operators to the right and all the annihilation operators to the left one gets $A^2/|z_+ - z_-|^2$. Here A is a constant equal to

$$\int dx \left((1+x)^2+\delta^2\right)^{-1/2} \left((1-x)^2+\delta^2\right)^{-1/2}.$$

This means that the most probable is the particle configuration where $|z_+ - z_-| \sim d$ (to get the physical parameter one should replace, as usual, δ by d). That is the right and left electrons with the opposite spins are in fact always near one another forming a chiral complex. In this sense the chiral fours are point-like entities as the $R\overline{L}$-pairs for one component fermions. In the same way one can give estimates of the distances between all the particles belonging to a four-particle complex.

The same calculation for a neutral four-particle complex are a little more cumbersome. The state with a single neutral pair is described by the wave function

$$|\Phi_0> = \mathrm{Tr}\left[\hat{a}^\dagger_{R,-}(x_-)\,\hat{b}^\dagger_{R,-}(x'_-)\,\hat{a}^\dagger_{L,+}(y_+)\,\hat{b}^\dagger_{L,+}(y'_+)\right.$$
$$\left.(x'_- - x_- - i\delta)^{-1}(y_+ - y'_+ - i\delta)^{-1}\frac{\sqrt{(x_- - y_+ + i\delta)(x'_- - y'_+ + i\delta)}}{\sqrt{(x_- - y'_+ + i\delta)(x'_- - y_+ + i\delta)}}\right]|F>. \tag{A2}$$

In order to find the characteristic size of a neutral complex consider the matrix element

$$<\Phi_0|\hat{a}^\dagger_{R,-}(z_-)\,a_{R,-}(z_-)\,\hat{a}^\dagger_{L,+}(z_+)\,\hat{a}_{L+}(z_+)\,|\Phi_0>$$

It is equal to

$$\mathrm{Tr}'\left[(\tilde{x}'_- - \overline{\tilde{x}'}_- - i\delta)^{-1}(\overline{\tilde{y}'}_+ - \tilde{y}'_+ - i\delta)^{-1}(\overline{\tilde{x}'}_- + i\delta)^{-1}(-\overline{\tilde{y}'}_+ + i\delta)^{-1}\right.$$
$$\frac{\sqrt{(z_- - z_+ - i\delta)(z_- - z_+ + \overline{\tilde{x}'}_- - \overline{\tilde{y}'}_+ - i\delta)}}{\sqrt{(z_- - z_+ - \overline{\tilde{y}'}_+ - i\delta)(z_- - z_+ + \overline{\tilde{x}'}_- - i\delta)}}$$
$$\left.(\tilde{x}'_- - i\delta)^{-1}(-\tilde{y}'_+ - i\delta)^{-1}\frac{\sqrt{(z_- - z_+ + i\delta)(z_- - z_+ + \tilde{x}'_- - \tilde{y}'_+ + i\delta)}}{\sqrt{(z_- - z_+ - \tilde{y}'_+ + i\delta)(z_- - z_+ + \tilde{x}'_- + i\delta)}}\right] \tag{A3}$$

(Tr' implies that one should integrate over all the variables besides z_α). The exact expression for this matrix element for arbitrary values of $\Delta Z = z_+ - z_-$ is rather cumbersome and non-informative. It is sufficient to prove that the most probable is the particle configuration where $\Delta Z \sim d$.

To do this let us note that for $\Delta Z \sim \delta$ all the integrals in Eq. (a3) converge and are dominated by the regions of the order δ. Let us show that for $\Delta Z \gg \delta$ the matrix element (A3) has an additional small factor $\delta/\Delta Z$. Consider, for instance, the integration over $\tilde{x}'_-$. Only the factor

$$\frac{\sqrt{(z_- - z_+ + \tilde{x}'_- - \tilde{y}'_+ + i\delta)}}{\sqrt{(z_- - z_+ + \tilde{x}'_- + i\delta)}}. \tag{A4}$$

in the integrand has a singularity in the lower semiplane. The rest integrand does not depend of ΔZ and has singularities only in the upper semiplane. In the main approximation in $\delta/\Delta Z$ (A4) tends to 1. ($\Delta Z \gg \delta$, while the regions $\tilde{x}'_-, \tilde{y}'_+$, giving the main contribution into the integral $\sim \delta$). Hence in this approximation all the integral (A3) vanishes. It is nonzero only in the next approximation due to the factors of the sort $\sqrt{(\tilde{x}'_- + i\delta)/(z_- - z_+)} \ll 1$. Thus the probability to find the electrons we are interested in at a large distance is small. Most probable is a four where these particles are at the distances $\sim d$.

---

[1] J. Voit, Rep. Prog. Phys. **57**, 977 (1994).

[2] V. J. Emery, in *"Highly Conducting One-Dimensional Solids*", ed. J. T. Devreese et al. (Plenum, New York, 1979), p.327.
[3] H. J. Schulz, Phys. Rev.Let. **71**, 1864 (1993).
[4] V.V. Afonin and V.Yu. Petrov, cond-mat/0407351
[5] A. A. Odintsov and Hideo Yoshika, Phys. Rev.B **59**, R10457 (1999).
[6] R. Egger and A. O. Gogolin, Phys. Rev. Let. **79**, 5082 (1997).
[7] C. Kane, L. Balents, and M. P. A. Fisher, Phys. Rev. Let. **79**, 5086 (1997).
[8] G't Hooft, Phys. Rev. D **14** p.3432 (1976)
[9] A.V. Smilga, Phys. Rev. D **46**, 5598 (1992)
[10] G.D.Machan, *"Many Particle Physics"* Plenum Press, New York, (1993).
[11] J. Hubbard, Phys. Rev. Let. **3**, 77 (1959).
[12] Alexei M.Tsvelik, *"Quantum Field Theory in Condenced Matter Physics",* Cambridge, University Press (1998).
[13] See, for example, Slavnov A. A, Faddeev L. D. *"Introduction to the quantum field theory"*. Moskow, "Nauka" (1978). This theorem is, in fact, a purely combinatorial statement. In the field theory we apply it mostly to Green functions. In statistical physics it is known as the first Mayer's theorem (Uhlenbek G.E., Ford G.W. and Montroll E.W. *"Lectures in statistical mechanics,"* American Mathematical Society, Providence (1963)).
[14] Note that the analogy with a bound state is quite limited. It would be more correct to write about a correlation in the momentum space.
[15] L. D. Landau, E. M. Lifshitz, *"Statistical Physics"*, Pergamon, 1986.
[16] This means that even in the strong interaction limit the spin phase can exist as a Kosterlitz — Thouless phase with the Thouless constant of the order of 1/2. We assign the chirality +1 to a right electron and a left hole and −1 to their counterparts.
[17] H. Ajiki and T. Ando, J. Phys. Soc. Jpn. **62**, 1255 (1993).
[18] A. Jorio, R. Saito, C. M. Lieber, M. Hunter, T. McClure, G. Dresselhaus, and M. S. Dresselhaus, Phys. Rev. Lett. **86**, 1118 (2001).
[19] C.Kane, L.Balents, and P.A.Fisher Phys. Rev. Let. **79**, 5086 (1997).